BRIEF COMMUNICATIONS

POLYTROPIC MODELS IN THE RELATIVISTIC GENERALIZED THEORY OF GRAVITATION

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Polytropic models were considered in [1] in the nonrelativistic approximation to the generalized theory. Below we report some results obtained for polytropic models with the equation of state

$$P = A\rho^{1+1/n} \quad (n \neq \infty) \tag{1}$$

in the relativistic generalized theory of gravitation. This theory is described in [2] and [3], in which the interior problem for a static spherically symmetric configuration is formulated, and the incompressible fluid model (n = 0) is studied. We will use the notation of [3] and assume that the reader is familiar with the content of that paper.

The quantity A in Eq. (1) is called the polytropic temperature of the model. We will not consider the choice of this temperature and merely note that a model with $A = A_*$ will be transformed to another model with $A = \alpha A_*$ (but with the same index n) under the following similarity transformation:

$$s \to \alpha s_*, \quad \rho \to \alpha^{-n}\rho_*, \quad P \to \alpha^{-n}P_*,$$
$$E \to E_*, \quad f \to f_*, \tag{2}$$

where

$$w = w_*, \quad M = \alpha^{n/2}M_*,$$
$$R = \alpha^{n/2}R_*, \quad P_0 = \alpha^{-n}P_{0*}. \tag{3}$$

Numerical integration was therefore carried out for $A_* = 1$, and Fig. 1 and Table 1 show the integral parameters

$$p_0 = P_0A^n, \quad M_* = MA^{-n/2},$$
$$R_* = RA^{-n/2}, \quad w = \frac{M_*}{R_*} = \frac{M}{R} \tag{4}$$

for the most important polytropics with n = 1.5 and 3. For comparison, Fig. 1 shows the function $M_*(p_0)$ for an incompressible fluid (ρ = 1).

For small values of p_0 (small w) both the Einstein theory and the corresponding nonrelativistic approximations (for still smaller p_0) are valid. The region $p_0 \sim 1$ (large w) corresponds to gravitars. The maximum admissible values of the central "pressure" p_0 are 2.070 (n = 0), 1.372 (n = 1.5), and 0.734 (n = 3). The saturation of the central pressure, which is clear from Fig. 1, is the result of a superposition of effects associated with the curvature of space and the weakening of gravitational interaction associated with large mass concentrations.

Table 1

The Most Important Parameters of Polytropic Models in the Relativistic Generalized Theory of Gravitation

n = 1.5				n = 3			
w	p_0	M_*	R_*	w	p_0	M_*	R_*
0.652	1.372	$1.12\cdot10^4$	$1.71\cdot10^4$	0.571	0.734	104.9	$1.83\cdot10^2$
.631	1.372	$2.79\cdot10^3$	$4.43\cdot10^3$	0.568	0.734	85.16	$1.49\cdot10^2$
.610	1.372	$6.97\cdot10^2$	$1.14\cdot10^3$	0.565	0.734	64.41	$1.14\cdot10^2$
.583	1.372	87.59	$1.50\cdot10^2$	0.562	0.734	52.28	93.01
.564	1.372	21.81	38.66	0.553	0.734	26.19	47.31
.555	1.370	10.90	19.64	0.546	0.734	13.16	24.10
.546	1.368	5.458	9.998	0.541	0.734	6.691	12.37
.535	1.363	2.733	5.106	0.515	0.729	3.285	6.378
.521	1.328	1.380	2.652	0.473	0.686	1.677	3.548
.490	1.212	0.715	1.459	0.356	0.555	0.890	2.503
.416	0.880	0.407	0.978	0.125	0.205	0.677	5.416
.264	0.140	0.321	1.217	$7.76\cdot10^{-2}$	$5.62\cdot10^{-4}$	1 963	25.29
.220	$4.61\cdot10^{-2}$	0.332	1.511	$5.62\cdot10^{-2}$	$3.64\cdot10^{-5}$	2.770	49.30
.155	$7.64\cdot10^{-3}$	0.324	2.088	$3.06\cdot10^{-2}$	$1.19\cdot10^{-6}$	3.619	$1.18\cdot10^2$
.037	$7.18\cdot10^{-5}$	0.149	4.010	$1.75\cdot10^{-2}$	$8.50\cdot10^{-8}$	3.911	$2.23\cdot10^2$
.012	$4.48\cdot10^{-6}$	$6.78\cdot10^{-2}$	5.473	$9.00\cdot10^{-3}$	$5.22\cdot10^{-9}$	4.126	$4.58\cdot10^2$
.003	$1.37\cdot10^{-7}$	$2.35\cdot10^{-2}$	7.833	$2.10\cdot10^{-3}$	$1.30\cdot10^{-11}$	4.216	$2.04\cdot10^3$

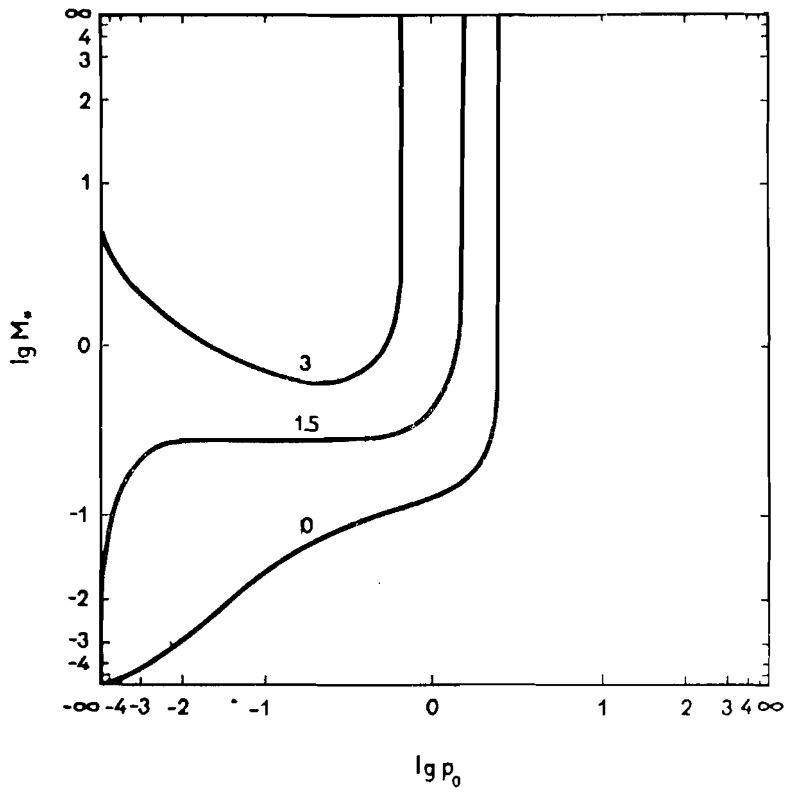

Fig. 1. The mass M_* as a function of central pressure p_0 (see Eq. (4)) for polytropic models with indices indicated on the curves.

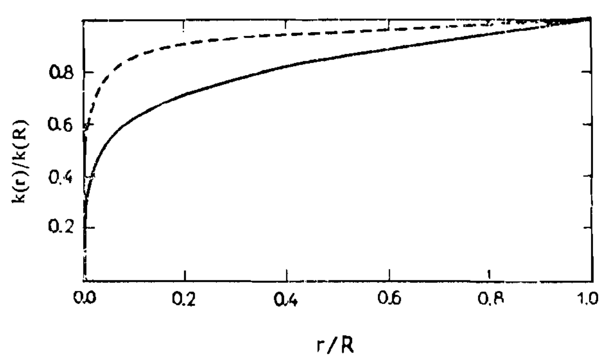

Fig. 2. Behavior of the gravitational scalar k(r) inside the gravitar.

The quantity

$$q = \frac{P}{\rho}, \tag{5}$$

is called the local relativistic parameter, and at the center of the model it is given by

$$q_0 = p_0^{n+1}. \tag{6}$$

The fact that the maximum possible values of q_0 are close to unity (see Table 2) is consistent with the

Table 2

Maximum Possible Values of $q_0 = P_0/\rho_0$ for Polytropic Models in the Generalized Theory of Gravitation

n	max q_0	
	Nonrelativistic theory	Relativistic theory
0	1.51	2.070
1.5	1.18	1.135
3	1.02	0.925

fundamental conclusion of statistical physics that $P/\rho < 1$.

Since the local condition $q_0 \sim 1$ overlaps (Table 1) the integral condition w ($w \gtrsim 0.5$), we must conclude that, in the generalized theory, the validity of any particular approximation is uniquely determined by the degree of compactness $w = M/R$ of the configuration.

We do not reproduce here the values of the gravitational mass defect, since its order of magnitude can be deduced from the example of the incompressible fluid [3] and the behavior of the functions $\lambda(r)$ and $\nu(r)$ inside the mass distribution. The values of the latter on the surface are given by the exterior Heckmann solution, whereas at the center of the configuration $\lambda(0) = -\infty$, $\nu(0) = \nu(R) - 2(n + 1) \ln (q_0 + 1)$.

The dashed line in Fig. 2 represents the quantitative variation of the gravitational scalar k(r) inside the configuration. For low w it touches the left-hand and upper side of the frame, whereas as w increases it asymptotically approaches the solid line. This limiting behavior for $w \to \infty$ is also characteristic for the other functions. Moreover, the asymptotic behavior is not very dependent on the polytropic index n. The function shown in Fig. 2 refers to n = 1.5. The corresponding functions for n = 0 and n = 3 lie below and above it, but this cannot be shown on the scale of our figure because they lie too close to it.

It is important to note that the region of distances in which there is an appreciable deviation of the interior solutions from those corresponding to the general theory of relativity is of the order of the gravitational radius $R_g = 2M$ of the configuration.

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REFERENCES

1. M. A. Mnatsakanyan and R. M. Avakyan, Astrofizika [Astrophysics], 4, 646, 1968.

2. G. S. Saakyan and M. A. Mnatsakanyan, Astrofizika [Astrophysics], 4, 567, 1968.

3. G. S. Saakyan and M. A. Mnatsakanyan, Astrofizika [Astrophysics], 5, 569, 1969.

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