BRIEF COMMUNICATIONS

SALMONA'S PAPER [6] ON MODELS OF STATIC CONFIGURATIONS

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Theoretical studies [1–5] within the framework of the generalized relativistic theory of gravitation have shown that it is possible, at least in principle, to construct models of static configurations whose masses are exceedingly large in comparison with the solar mass. This possibility arose because the exterior Heckmann solution (solution of the field equation in empty space for a point mass in the generalized theory) has no singularity. On the other hand, Salmona [6] has considered a model of a neutron star, using the field equations and the exterior solution obtained by Dicke and Brans [7–8]. His results are not very different from the Einstein theory and, in principle, indicate that the generalized theory excludes the possibility of static bodies with masses substantially exceeding the solar mass. The aim of the present paper is to remove the resulting contradiction with the work of Saakyan and Mnatsakanyan by indicating that the analysis given by Salmona is not correct.

The variational principle and the tensor field equation resulting from it in [1] and [6–8] are essentially identical apart from the notation. Dicke and Brans considered the equations for a spherically symmetric static field in an isotropic coordinate system with the line element

$$ds^2 = -e^{2\alpha(\rho)}dt^2 + e^{2\beta(\rho)}(d\rho^2 + \rho^2 d\Omega^2), \quad (1)$$

and found the solution in empty space for a point mass M in the following form

$$e^{\alpha} = e^{\alpha_0}[(1 - B/\rho)/(1 + B/\rho)]^{1/\lambda},$$
$$e^{\beta} = e^{\beta_0}[(1 + B/\rho)^2[(1 - B/\rho)/(1 + B/\rho)]^{(\lambda - C - 1)/\lambda},$$
$$\Phi = \Phi_0[(1 - B/\rho)/(1 + B/\rho)]^{C/\lambda},$$

where α_0, β_0, Φ_0, C, λ and B ~ M are constants. This solution was used by Salmona as the exterior solution for the static model, and would appear to have a singularity at the distance ρ = B from the point mass.

In actual fact, however, the point mass itself corresponds to the value ρ = B (and not ρ = 0). With increasing distance from the center of the field the variable ρ varies in the range

$$B \leqslant \rho < \infty. \quad (3)$$

This very important fact appears to be ignored by Dicke and Brans and by Salmona.

The coordinate distance from the center of the field is the quantity r in the line element written in the form

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2 d\Omega^2. \quad (4)$$

When this metric is employed, the solution in empty space for a point mass is, in fact, the Heckmann solution [1] which is regular for all r in the range (0, ∞). The function $e^{\lambda(r)}$ vanishes at the center of the field. If we transform from (4) to (1), the Heckmann solution becomes identical with (2), but the range of variation of the new variable ρ is restricted by the condition $\rho \geq B$. This is connected with the fact that $e^{\lambda(r)} \to 0$ as $r \to 0$. In fact, the relation between r and ρ is given by

$$\rho = B \exp\left|\int_0^r \frac{e^{\lambda(r)/2}}{r}\,dr\right|, \quad (5)$$

where B = const ≠ 0. Since $e^{\lambda(r)} \to 0$ as $r \to 0$, the formula given by (5) transforms the center of the field r = 0 to ρ = B.

The transformation formula (5) is valid both outside and inside the mass distribution. It is shown in [2] that the field equation for the interior admits the following behavior for the function $e^{\lambda(r)}$ at the center of the model: (a) $e^{\lambda(r)}$ vanishes as $r \to 0$, or (b) $e^{\lambda(0)}$ is a nonzero constant. In the former case the center of the star corresponds to ρ = B. In the second case, it is clear from (5) that ρ = 0 at the center. Moreover, since the exterior solution does not admit ρ < B, the coordinate radius of the configuration has a definite lower bound which is proportional to the mass of the star. It thus becomes clear why the second possibility excludes the existence of massive configurations. On the other hand, the first possibility admits models of static bodies with exceedingly high masses. In his analysis of the behavior of the interior solutions, Salmona assigns ρ = 0 to the center of the model and, therefore, case (a) is automatically excluded from the analysis.

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REFERENCES

1. G. S. Saakyan and M. A. Mnatsakanyan, Astrofizika [Astrophysics], 4, 567, 1968.

2. G. S. Saakyan and M. A. Mnatsakanyan, Astrofizika [Astrophysics] (in press).

3. G. S. Saakyan and M. A. Mnatsakanyan, Abstracts of Papers Read at the Fifth International Conference on Gravitation and Relativity Theory [in Russian], Tbilisi, p. 198, 1968.
4. M. A. Mnatsakanyan (in press).
5. G. S. Saakyan and M. A. Mnatsakanyan, Astrofizika [Astrophysics] (in press).
6. A. Salmona, Phys. Rev., 154, 1218, 1967.
7. C. H. Brans and R. H. Dicke, Phys. Rev., 124 925, 1961.
8. C. H. Brans, Phys. Rev., 125, 2194, 1962.

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