VELOCITY DISTRIBUTION OF O-B STARS IN ASSOCIATIONS

L. V. Mirzoyan and M. A. Mnatsakanyan

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The velocity distribution of O-B stars in a synthetic association has been investigated. A method is presented for the determination of the mean velocity of stars in spherically symmetric systems at different distances from the center, using their residual radial velocities and the distribution projected on the celestial sphere. By applying this method to the synthetic association it is shown that the dependence of the mean space velocity of the stars on the distance from the center of the system is a linearly increasing function. Possible interpretations of this result are discussed. The only interpretation not in conflict with observational data (distribution of stellar density in associations, stellar masses, and so on) is that based on the assumption of expanding associations. This leads to the conclusion that the linearly increasing function v(r) can be regarded as important evidence for the expansion of associations and indicates that most known stellar associations are dynamically unstable.

INTRODUCTION

Internal motions of O-B stars in associations, which are largely due to velocities given to them during their formation [1], are a source of information on the stellar formation process. Even the early studies of the intrinsic motions of stars in the nearest O-associations [2, 3] confirm the theoretically predicted expansion of such systems [1]. However, owing to the large distances of the O-associations, the internal motions of O-B stars belonging to them are usually small and their determination is subject to substantial relative errors. This is why motions of O-B stars can usually be investigated only by considering their radial velocities.

However, it is shown in [4] that data on the residual radial velocities of O-B stars definitely suggest the expansion of stellar associations. This conclusion was based on a statistical study of the distribution of residual radial velocities of O-B0 stars in a synthetic association constructed by superimposing subsystems of such stars around the nuclei of stellar associations.

It was found that the dispersion of the residual radial velocities of the stars and the mean value of their absolute magnitude increase with distance from the center of the synthetic association, and this can be regarded as a consequence of continuous formation and escape of stars with different velocities from the nuclei of the associations, which are the stellar-formation centers. The observed distribution of stars around the center of a synthetic association is in good agreement with ideas on the time-independent nature of the flow of stars from the center for the entire synthetic association, i.e., the time independence of the set of stellar associations with respect to the star-formation process.

In this paper we report the results of a study of the velocity distribution of O-B stars in a synthetic association, based on their residual radial velocities and the projected distribution on the celestial sphere, using the new method described below.

FORMULATION OF THE PROBLEM

Consider an expanding stellar association with a spherically symmetric distribution of stars around the center. The large dispersion of velocities with which the stars are emitted from the generating nucleus

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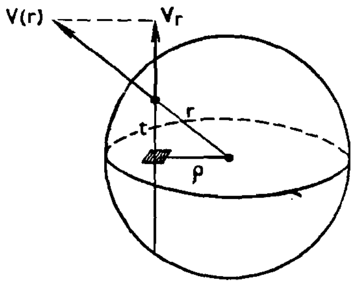

Fig. 1

means that each spherical shell (r, r + dr) centered on the neucleus will always contain stars with different velocities. Thus, young stars having low velocities cannot attain a large distance from the nucleus (their lifetime has been insufficient) and hence in an expanding system of young stars of given age the mean velocity must increase with increasing distance r from the center.

We shall suppose that the velocities of all the stars are radial with respect to the center of the association, and will use v(r) to denote the mean-absolute velocity relative to the center for stars in a spherical layer (r, r + dr). It will be shown later that this assumption is not a restricting one. The conclusions about the behavior of the function v(r), which are based on this assumption, turn out to be valid even under more general assumptions about the direction of the velocity.

Our problem is to determine the function v = v(r), i.e., the dependence of the mean velocity of expansion of the association on the distance from its center.

The basic data will be the (observed) distribution of association stars projected onto the celestrial sphere, and their radial velocities relative to the center of the association.

Let $w(\rho)$ be the sum of the absolute magnitudes of the observed residual radial velocities of stars per unit area at a distance ρ from the center of the association in projection onto the celestial sphere.

It is clear from Fig. 1 that

$$w(\rho) = 2\int_0^\infty v_r \Phi(r)\, dt, \tag{1}$$

where $\Phi(r)$ is the spatial density of stars in the association at a distance r from its center, and v_r is the mean-residual-radial velocity. If we substitute

$$v_r = v(r)\frac{t}{r}$$

in Eq. (1) and change the integration variable in accordance with the formula

$$t = \sqrt{r^2 - \rho^2},$$

we obtain the following relation between w and v:

$$w(\rho) = 2\int_\rho^\infty v(r)\Phi(r)\, dr. \tag{2}$$

Differentiating Eq. (2) with respect to ρ, and replacing ρ with r, we obtain

$$v(r) = -\frac{1}{2\Phi(r)}\frac{dw(r)}{dr}. \tag{3}$$

The function $w(\rho)$ can be determined from the formula

$$w(\rho) = \frac{1}{2\pi\rho}\frac{dW(\rho)}{d\rho}, \tag{4}$$

where $W(\rho)$ is the sum of the absolute magnitudes of residual radial velocities deduced from observations, for stars lying in a circle of radius ρ which is centered on the nucleus of the association in projection onto the celestial sphere.

The stellar density $\Phi(r)$ is usually calculated from the two-dimensional distribution of stars on the celestial sphere [5], or from the one-dimensional distribution F(x), using the formula [6]

$$\Phi(r) = -\frac{1}{2\pi r}\frac{df(r)}{dr}. \tag{5}$$

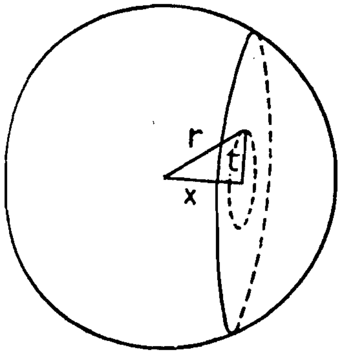

Fig. 2

In this expression

$$f(x) = -\frac{dF(x)}{dx}, \tag{6}$$

and F(x) is the number of stars in the band (x, ∞), i.e., in the half-plane whose boundary is at a distance x from the center of the association.

Equation (3) can, at least in principle, provide the solution of the problem, namely, the function v(r). However, the accuracy with which this formula can be used in practice is not very high because it involves the second derivatives of the observed functions F(x) and $W(\rho)$.

The problem is substantially simplified if we restrict our attention to the study of the qualitative behavior of the function v(r). It turns out that it is then sufficient to determine $\bar{v}$ as a function of $\bar{r}$, where $\bar{v}$ is the mean-spatial velocity and $\bar{r}$ is the mean distance from the center of the system for stars belonging in a definite fashion to a chosen grouping.

DERIVATION OF BASIC FORMULAS

Consider a star belonging to an association, this star being located in a plane parallel layer of unit thickness at a distance x from the center of the association (Fig. 2). The number of stars in the layer is $f(x)$, and the sum of their distances from the center

$$\sum_{x}^{x+1} r = 2\pi \int_{0}^{\infty} r\Phi(r)\, t dt = 2\pi \int_{x}^{\infty} r^2 \Phi(r)\, dr, \tag{7}$$

i.e., it is equal to one-half of the number of stars in the association outside a sphere of radius x. The symbol $\sum_{x_1}^{x_2}$ represents the sum of the corresponding quantities for stars in the band (x_1, x_2).

Using Eqs. (5) and (6), we can write

$$\sum_{x}^{x+1} r = F(x) - x\frac{dF(x)}{dx}. \tag{8}$$

Here and henceforth we are assuming that

$$f(\infty) = 0,\ F(\infty) = 0,\ W(0) = 0,\ W(\infty) = \text{const},\ w(\infty) = 0. \tag{9}$$

The sum of the absolute magnitudes of the space velocities of stars in the above layer (we are concerned with the space velocity relative to the center of the association) is given by

$$\sum_{x}^{x+1} v = 2\pi \int_{0}^{\infty} v(r)\, \Phi(r)\, t dt = 2\pi \int_{x}^{\infty} v(r)\, \Phi(r)\, r dr, \tag{10}$$

and if we use Eq. (3) we can express this in terms of the function $w(\rho)$:

$$\sum_{x}^{x+1} v = -\pi \int_{x}^{\infty} r\frac{dw(r)}{dr} dr. \tag{11}$$

Integrating this by parts, and recalling Eq. (4), we obtain

$$\sum_{x}^{x+1} v = \pi \int_{x}^{\infty} w(r)\, dr + \pi x w(x) = \frac{1}{2}\int_{x}^{\infty} \frac{1}{r}\frac{dW(r)}{dr} + \frac{1}{2}\frac{dW(x)}{dx}$$

$$= \frac{1}{2}\left[\int_{x}^{\infty} \frac{W(r)}{r^2} dr - \frac{W(x)}{x} + \frac{dW(x)}{dx}\right]. \tag{12}$$

Let us now determine the sums of distances and of space velocities of stars in the layer (x, ∞). By definition, the number of stars in the layer (x, ∞) is equal to $F(x)$.

Integrating Eqs. (8) and (12) with respect to x between x and ∞, we obtain

$$\sum_x^{\infty} r = \int_x^{\infty}\left(\sum_x^{x+1} r\right)dx = 2\int_x^{\infty} F(x)\,dx + xF(x) \tag{13}$$

for the sum of distances, and

$$\sum_x^{\infty} v = \int_x^{\infty}\left(\sum_x^{x+1} v\right)dx$$
$$= \frac{1}{2}\left[\int_x^{\infty}\int_z^{\infty}\frac{W(\rho)}{\rho^2}\,d\rho\,dz - \int_x^{\infty}\frac{W(\rho)}{\rho}\,d\rho + W(\infty) - W(x)\right] \tag{14}$$

for the sum of space velocities.

The corresponding sums and the number of stars in an arbitrary layer (x_1, x_2) are determined by the differences

$$\sum_{x_1}^{\infty} r - \sum_{x_2}^{\infty} r, \quad \sum_{x_1}^{\infty} v - \sum_{x_2}^{\infty} v, \quad F(x_1) - F(x_2). \tag{15}$$

If we divide the sum of distances and the sum of velocities by the number of stars in a given layer we obtain the mean values $\bar{r}$ and $\bar{v}$. By varying the thickness of the layer and its position in the association, we obtain a parametric dependence (x is the parameter) of $\bar{v}$ on $\bar{r}$ in the form $\bar{v} = \bar{v}(\bar{r})$.

Equations (13)-(15) can be used to express $\bar{v}(\bar{r})$ in terms of the observable functions $F(x)$ and $W(\rho)$ and the integrals of them. It will be seen from the ensuing analysis that the function $\bar{v}(\bar{r})$ gives definite information about the behavior of the required function $v(r)$. For example, it can be shown that if $\bar{v}(\bar{r})$ is an increasing function, the function $v(r)$ is also an increasing function.

It is important to note that the range of variation of $\bar{r}$ for all the possible changes in x_1 and x_2 is restricted from below in accordance with Eq. (7) by the quantity

$$\bar{r}_{\min} = \frac{F(0)}{f(0)}, \tag{16}$$

which is the mean distance from the center for stars in an infinitely thin layer passing through the center of the association.

To determine $\bar{v}(\bar{r})$ in the region $\bar{r} < \bar{r}_{\min}$ we can consider other groupings of stars, for example, stars lying inside spheres of different radius r. The mean distances and velocities can then be calculated from the functions $F(x)$ and $W(\rho)$ and their first derivatives. We shall not reproduce the corresponding expressions since we shall not need them. The point is that the density $\Phi(r)$ on which $\bar{r}_{\min}$ depends is found to increase very substantially toward the center of the synthetic association. Hence, the value of $\bar{r}_{\min}$ is quite small.

DETERMINATION OF THE FUNCTION $\bar{v}(\bar{r})$

To determine $\bar{v}(\bar{r})$ we have used the radial velocities of 290 O-B1 stars from Wilson's catalog [7]. The distribution of these stars around the corresponding nuclei was determined from their distances from the nearest nuclei in projection on the celestial sphere. The synthetic association was constructed on the basis of data taken from the Ruprecht catalog [8] and the stellar distances were taken from the Hiltner list [9]. The observational material consists of the absolute values of the residual radial velocity (corrected for the motion of the centers of the associations and the motion of the sun) and the distances of stars from the centers of the associations projected on the celestial sphere and shown in Fig. 3. For most stars (about 100) in the neighborhood of $\rho < 50$ pc from the center of the association the distances ρ_i are very un-

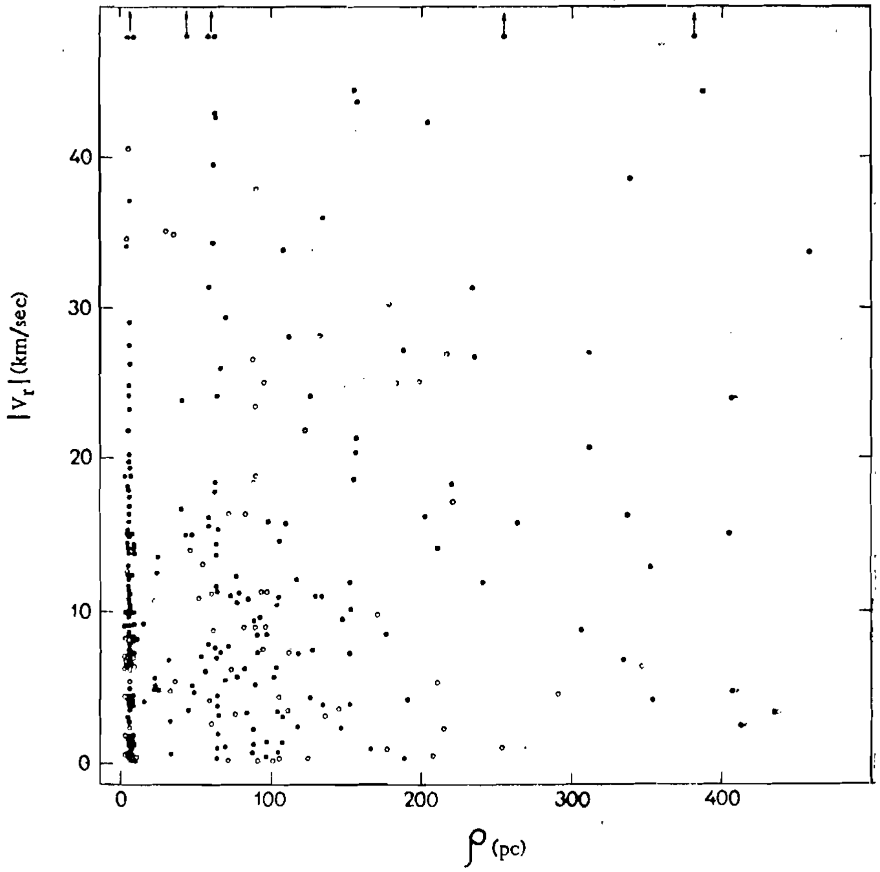

Fig. 3. Observational data used for the 290 O-B1 stars: ρ_i is the distance from the center of the synthetic association projected onto the celestial sphere, and $|v_r|$ is the absolute magnitude of the residual radial velocity. The dots represent O-B0 stars; the open circles represent B0.5-B1 stars. The distances are uncertain for 102 stars with $\rho < 50$ pc. The radial velocities of these stars are placed on the left of the figure.

certain. We have placed them on the left of Fig. 3 to indicate their radial velocities. The values of ρ_i for these stars were assumed to be zero in our calculations.

In the numerical solution of the problem we assumed that the radius R of the association was finite and have transformed Eq. (14) accordingly. Thus, the integral $\int_x^\infty$ was replaced by the sum $\int_x^R + \int_R^\infty$, and the repeated integral was written in the form

$$\int_x^\infty \int_z^\infty = \int_x^R \int_z^R + \int_x^R \int_R^\infty + \int_R^\infty \int_z^\infty \tag{17}$$

so that, bearing in mind that

$$\text{for} \quad \rho > R \quad W(\rho) = W(\infty) = \text{const} = W(R), \tag{18}$$

the sum of the space velocities is now given by

$$\sum_x^R v = W(R) + \frac{1}{2}\left[\int_x^R \int_z^R \frac{W(\rho)}{\rho^2} d\rho dz - \int_x^R \frac{W(\rho)}{\rho} d\rho - \frac{W(R)}{R} x - W(x)\right] \tag{19}$$

TABLE 1

Star	No. of stars	a, km/sec • kpc	b, km/sec
O–B0	222	102.56	18.34
B0.5–B1	68	37.96	15.83
O–B1	290	88.94	17.31

which replaces Eq. (14). We note that the formal substitution $R = \infty$ in Eq. (19) is not correct and does not lead to Eq. (14). The introduction of the function $W^*(\rho) = W(R) - W(\rho)$ will show the identity of Eqs. (14) and (19).

We determine the function F(x) by considering the two-dimensional distribution of stars on the celestial sphere. If ρ_i is the projected distance of the i-th star from the center of the association, the number of stars in the range (x, R) is given by [10]

$$F(x) = \frac{1}{\pi} \sum_{\rho_i > x} \arc\cos \frac{x}{\rho_i}. \tag{20}$$

Substituting Eq. (20) in Eq. (13), and integrating by parts, we obtain

$$\sum_x^R r = \frac{2}{\pi} \sum_{\rho_i > x} \sqrt{\rho_i^2 - x^2} - xF(x). \tag{21}$$

It is useful in calculations to remember that

$$\sum_0^R r = \frac{2}{\pi} \sum_i \rho_i, \quad \sum_0^R v = W(R). \tag{22}$$

The function $\bar{v}(\bar{r})$ is determined by calculating the mean quantities $\bar{v}$ and $\bar{r}$ for layers of finite thickness (0, x), where x is varied between x = 0 and x = R = 500 pc with a step $\Delta x = 20$ pc. (We assume that the radius of the synthetic association is 500 pc, but the result will not be affected if we take an arbitrary value R > 500 pc. Since at such distances the function F(x) is negligible, and $W(\rho)$ is constant, Eqs. (19)-(21) do not depend on the numerical value of R if it exceeds 500 pc.) The quantity $\bar{r}$ was varied between $\bar{r}_{min} = 30$ pc and 100 pc. We then considered the finite layer (x, R), where the variable x was also varied with a step of 20 pc. This yielded the function $\bar{v}(\bar{r})$ for the region 170 pc < $\bar{r}$ < 500 pc. The region 100 pc < $\bar{r}$ < 170 pc was absent because of the uncertainty in ρ_i for most stars with ρ less than 50 pc. For this region we have considered only the two layers (20, 100 pc) and (40, 140 pc), using Eq. (15) with mean distances of the corresponding stars $\bar{r}$ = 132 pc and 145 pc. The integrals in Eq. (19) were evaluated by the trapezium formula with a step $\Delta x = 20$ pc. We note that the function $\bar{v}(\bar{r})$ in the intervals 170-500 pc and 100-170 pc is, in principle, completely determined by the values in the range 30-100 pc. We are presenting the calculations in these intervals only to obtain the best representation of the behavior of this function.

The results of calculations referring to all the O-B1 stars in the synthetic association (290 stars) and individually to stars belonging to the spectral types O-B0 (222) and B0.5-B1 (68) are shown graphically in Fig. 4. We note that the function $\bar{v}(\bar{r})$ obtained with the aid of Eqs. (16)-(21), even for the small number of stars used, is practically continuous. Figure 4 shows only the discrete values of this function.

In all three cases, $\bar{v}(\bar{r})$ is an increasing function. (It is important to note that a sufficient but not necessary condition for the increase in $\bar{v}(\bar{r})$ is that the mean-radial velocity should not decrease with increasing distance from the center of the association when projected onto the celestial sphere; it is clear from Fig. 3 that this condition is satisfied from the O-B0 and B0.5-B1 stars which we are considering.) The straight lines in Fig. 4 were obtained by the method of least squares. The values of the constants a and b in the equations for the straight lines (i.e., $\bar{v} = a\bar{r} + b$) are shown in Table 1.

Therefore, the numerical calculations based on observational data show that the function $\bar{v}(\bar{r})$ for a synthetic association is linear to sufficient accuracy.

It can be shown (see Appendix I) that, in the special case when $\bar{v}(\bar{r})$ is a linear function, the required function v(r) is also linear and has the same parameters as $\bar{v}(\bar{r})$. In other words, in this case, the above

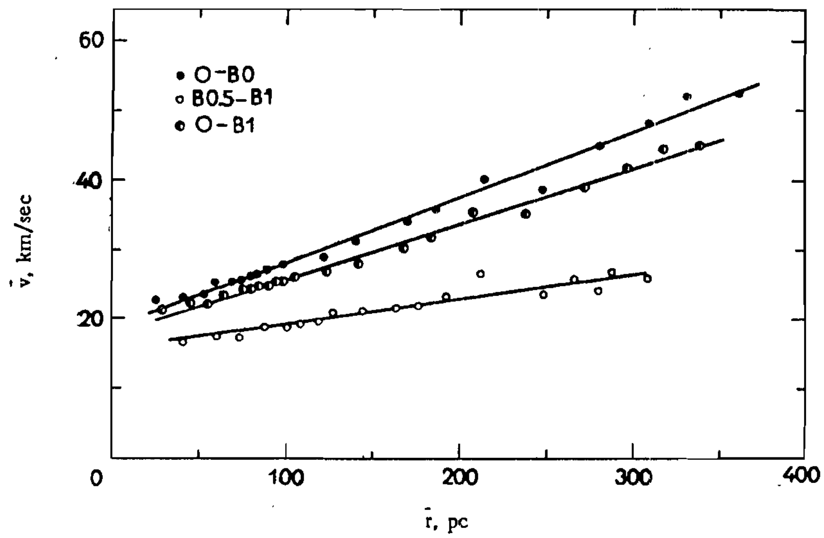

Fig. 4. The function $\bar{v}(\bar{r})$ in the synthetic association. The straight lines were obtained by the method of least squares. Because $\bar{v}(\bar{r})$ is linear, $v(r) \equiv \bar{v}(\bar{r})$ for all three cases.

functions must be identically equal: $v(r) \equiv \bar{v}(\bar{r})$. Although, in general, two groups of stars which, on the average, are equidistant from the center of the association will not have equal mean velocities, the linear function $\bar{v}(\bar{r})$ should remain unaltered independently of the method of forming the groups for which $\bar{v}$ and $\bar{r}$ are determined.

In this case, it also turns out that, because $\bar{v}(\bar{r})$ is linear in the region $\bar{r} > \bar{r}_{min}$, it must also be linear for $\bar{r} < \bar{r}_{min}$ (see Appendix II). Consequently, the results given above (Fig. 4) indicate that the dependence of $\bar{v}$ on $\bar{r}$ (and hence the dependence of v on r) in the synthetic association is a linearly increasing function for all $r > 0$.

We did not take into account ten stars in our calculations (in addition to the 290) for which ρ_i was greater than the arbitrarily assumed R = 500 pc. The inclusion of these stars in the calculations would lead to a change in the radial velocities of the centers of the corresponding associations by a few km/sec (these radial velocities were taken to be equal to the mean-radial velocity of the component stars) and, consequently, to a similar change in the residual radial velocities of the stars. However, the function $\bar{v}(\bar{r})$ obtained when these stars were included turned out to be in unexpectedly good agreement with Fig. 4. Hence, the method we have employed is not very sensitive to random errors in the radial velocities.

It is important to note the following fact. The distribution of the stellar samples observed in projection, which we have used to determine $\bar{v}$ and $\bar{r}$, may not be circularly symmetric. It is possible to artificially symmetrize the given distribution by rotating it about the center of the association through all possible angles, superimposing it on itself, and then taking an average over the number of rotations. This method was used to obtain the one-dimensional distribution of stars in the given sample [10], but the function $W(\rho)$ remained unaltered. The spherically symmetric distribution in space corresponding to the projected distribution obtained in this way will not, in general, be similar to the distribution of stars in the system under investigation as a whole. Moreover, it is possible, at least formally, to obtain even negative stellar densities. Since the derivation of $\bar{v}(\bar{r})$ is based only on the mean characteristics $\bar{v}$ and $\bar{r}$ of the stars, the correspondence between the space densities of stars in the given sample and all the stars in the system will not necessarily occur.

DIRECTIONS OF SPACE VELOCITIES

In deriving our basic formulas for the determination of the function v(r), we assumed that the space velocities of all the stars in the association were radial relative to the center of the system. It may turn out that this is a restriction on our analysis and, in fact, predetermines our conclusion about the increasing nature of the function v(r). It can be shown, however, that our conclusions remain in force even under more general assumptions with regard to the velocity directions in the system.

Consider stars at different distances from the center of the association (spherical shells). The above conclusion that the mean v increases with increasing r shows that the mean-absolute radial velocity v_r increases with increasing distance of the star from the center of the association. If this were not so, the function v(r) would not increase radially for the assumed radial direction of the velocities. Moreover, because the radial velocities of the stars in each concentric sphere are isotropic, we have

$$v_r(r) = \frac{1}{2} v(r). \tag{23}$$

Let us suppose that the directions of the stellar velocities in the system have an arbitrary distribution at each point. Assuming that the spatial velocities of stars located in any of the spheres have an isotropic spatial distribution, we have

$$u(r) = 2 v_r(r) = v(r). \tag{24}$$

In this equation u(r) is the mean-space velocity of stars in a thin spherical shell of radius r.

For example, Eq. (24) is satisfied when the stellar-velocity distribution for stars in the neighborhood of each point in the system is isotropic. It is also valid when the stellar velocities at each point in the system lie in a plane perpendicular to the radius drawn through this point, and in each such plane the velocities are distributed isotropically. This case corresponds to the motion of stars in randomly oriented circular orbits centered on the center of the system. It is clear that, in such cases, the mean-space velocity increases with increasing r in accordance with Eq. (24). The condition of isotropic distribution of stellar velocities in a given sphere means that the observed distribution of radial velocities does not depend on the direction of observation of the system.

The function u(r) increases with r even under the more general assumption that the distribution of the radial velocities of stars in each sphere depends on the direction of observation, but it does so in the same way for all concentric spheres, e.g., when the stellar motions take place in orbits whose planes are parallel to each other.

Thus, the conclusion that the function u(r) is linearly increasing is valid under more general assumptions about the directions of the stellar velocities in the system.

DYNAMIC IN- STABILITY OF ASSOCIATIONS

Consider the dynamic stability of associations in their own gravitational fields, using the increasing function u(r) which represents the mean-space velocity of stars as a function of their distance from the center of the synthetic association.

We must consider not the entire synthetic association, but the individual associations, because only stars belonging to a given association participate in the interaction. The function v(r) was obtained for the synthetic association and is valid only on the average. It can therefore be used only for the hypothetical "mean" association having characteristics close to the mean characteristics of all the possible associations.

To solve our problem let us consider the case of circular orbits. A star at a distance r from the center of the system can move on a circular orbit subject to the condition

$$u(r) = \left[\frac{\gamma M(r)}{r} \right]^{1/2}, \tag{25}$$

where M(r) is the total mass of stars in the association inside a sphere of radius r, and γ is the gravitational constant.

For the sake of simplicity, we assume that all the stellar masses are equal to m. The function M(r) can then be expressed in terms of the space density Φ(r):

$$M(r) = 4\pi m \int_0^r \Phi(r) r^2 dr. \tag{26}$$

It follows from Eqs. (25) and (26) that an increasing function u(r) is consistent with the assumption of stable circular orbits only when the spatial density Φ(r) decreases with distance more slowly than r^{-2}

(we note that the case of stable noncircular orbits differs from the case of circular orbits by the slower increase of u with r for given Φ (r)).

In particular, when Φ (r) = const inside the entire association we have

$$u(r) \sim r. \tag{27}$$

Although this assumption is in sharp conflict with the observational data on the stellar-density distribution in associations [11, 12], this example illustrates the basic possibility of a linear increase of u with r in dynamically stable systems.

Using Eq. (25), we can estimate the mass of the association in which stable motions of stars with velocities close to those obtained above for synthetic associations will be possible. For example, when u = 20 km/sec we have at r = 100 pc from the center of the system

$$M(r) = u^2(r)\, r/\gamma \approx 10^7 M_{\odot},$$

whatever the form of the function Φ(r).

The absence of such large masses from associations for linearly increasing u(r) leads directly to the conclusion of the dynamic instability of the "mean" association which we are considering, i.e., the set of the corresponding stellar associations on the average.

It is important to note that the mean-space velocities of stars calculated on the basis of the residual radial velocities may be too low because the radial velocity of the center of the system may, in fact, differ from the mean-radial velocities of the stars used in the calculations.

For the sake of completeness, we must note that the increase of u with r admits of one further (although extremely improbable) interpretation which is not connected with the expansion of stellar associations [4]. Let us suppose that there is a galactic background of O-B stars projected onto the given association, and the stars in this background have velocities much greater than the velocity of the members of the association with respect to the center of gravity of the association. The observed function u(r) can then be interpreted as the consequence of the different percentage content of O-B stars of these two types at different distances from the center of the association. In other words, large values of u at large distance from the center of the system are then due to the fact that the number of O-B stars in the system increases with distance and, conversely, there is an increasing number of projecting O-B stars in the general galactic background.

It is important to note, however, that if such a background were present it would be quite simple to take it into account in the calculations and eliminate it from our analysis.

CONCLUSIONS

Analysis of the residual radial velocities and the spatial distribution of 290 O-B1 stars in a synthetic association shows that the mean-space velocity increases with distance from the center of the association.

An analogous result was obtained for narrower spectral;type intervals, namely, when all the stars included in the calculation were divided into two groups, i.e., O-B0 (222 stars) and B0.5-B1 (68 stars). This is illustrated by Fig. 4 which shows the mean velocity of expansion as a function of the mean distance from the center of the association for all stars and for stars in the two groups noted above. Data referring to different samples are in qualitatively good agreement with each other. It also follows from these data that, on the average, the velocities of the O-B0 stars somewhat exceed the corresponding velocities of the B0.5-B1 stars.

Thus, the analysis of the residual-radial velocities of O-B1 stars in stellar associations, which is based on Eqs. (19)-(21), completely confirms our previous conclusion [4] that the mean-space velocity of these stars increases with distance from the center of the synthetic association. It follows from the above analysis that this fact is a very important argument in favor of the idea of expansion of stellar associations and their dynamic instability.

We are greatly indebted to V. A. Ambartsumyan for valuable discussions and to E. S. Kazaryan, and A. V. Terebizh for assistance in the calculations.

APPENDIX I

Linear Form of $v(r)$. Suppose that for different groups of stars (different position in space and number of stars) the function $\bar{v}(\bar{r})$ is linear, i.e.,

$$\bar{v} = a\bar{r} + b. \tag{I.1}$$

Both $\bar{v}$ and $\bar{r}$ depend on the region Ω of space occupied by the corresponding stellar group.

We assume that the volume and position of Ω vary continuously. If $n(\Omega)$ is the number of stars in Ω, then, by definition,

$$\bar{v} = \frac{1}{n}\int_{\Omega} v dn, \qquad \bar{r} = \frac{1}{n}\int_{\Omega} r dn.$$

By varying the region Ω we have

$$\frac{\delta\bar{v}}{\delta n} = \frac{v - \bar{v}}{n}, \qquad \frac{\delta\bar{r}}{\delta n} = \frac{r - \bar{r}}{n} \tag{I.2}$$

or

$$\frac{\delta\bar{v}}{\delta\bar{r}} = \frac{v - \bar{v}}{r - \bar{r}}. \tag{I.3}$$

Using Eq. (I.1) we can transform Eq. (I.3) to the form

$$\frac{\delta\bar{v}}{\delta\bar{r}} = \frac{v - a\bar{r} - b}{r - \bar{r}}.$$

Comparing this with the ratio $\delta\bar{v}/\delta\bar{r} = a$, which follows from Eq. (I.1), we obtain

$$v = ar + b.$$

The linear relation between $\bar{v}$ and $\bar{r}$ given by Eq. (I.1) is valid for all values of $\bar{r}$ which correspond to an arbitrary, continuously varying part of Ω. In particular, if the dimensions of this subregion are infinitely small, then Eq. (I.3) is valid for all values of r included in the averaging process defined by Eq. (I.1). The equivalent statement is proved in the next appendix for the special case in which Ω is a plane-parallel cut through the association.

APPENDIX II

Linear Form of $\bar{v}(\bar{r})$ for Small Values of $\bar{r}$. Let us suppose that $\bar{v}(\bar{r})$ is a linear function in the region $\bar{r} > \bar{r}_{min}$ for stars inside the plane-parallel layers of finite thickness which we have considered. As already noted, the linear from of $\bar{v}(\bar{r})$ persists even when the stellar groups are arbitrarily chosen for $\bar{r}$ between $\bar{r}_{min}$ and R. In particular, this is valid for stellar groups inside arbitrary disks of equal thickness, located at different distances from the center of the association. We discuss below disks of equal but infinitesimal thickness.

Consider stars inside a disk of arbitrary radius passing through the center of the association. Having determined for these stars the mean velocity and mean distance for different values of the radius of the disk less than R, we can deduce $\bar{v}(\bar{r})$ for the region $\bar{r} < \bar{r}_{min}$.

In fact, the number of stars inside a disk of radius x is the difference between the number of stars in a disk of radius R and in a flat ring (x, R) which is a continuation of the first disk up to the last.

On the other hand, it is readily shown that the number of stars in flat rings of equal thickness which lie between two given concentric spheres at an arbitrary distance from the center of the system is constant. In particular, the number of stars in the central flat ring (x, R) is equal to the number of stars inside the disk which is the cross section of the association at a distance x from its center. This is valid not only for the number of stars but also for the sum of any scalar quantities having a spatial distribution which is spherically symmetric with respect to the center of the system, and in particular, for the sum of spatial velocities and the sum of distances of stars from the center of the association.

Let the number of stars inside a disk of unit thickness at a distance x from the center of the association be denoted by $f(x)$, and the sum of the stellar velocities and distances by V(x) and P(x), respectively. From the condition $\bar{v} = a\bar{r} + b$, we have

$$\frac{V(x)}{f(x)} = a\frac{P(x)}{f(x)} + b. \tag{II.1}$$

If we write this for two values of the argument, namely, x and 0, we have

$$V(x) = aP(x) + bf(x), \qquad V(0) = aP(0) + bf(0). \tag{II.2}$$

As shown above, the mean-space velocity and the mean distance of stars inside a central disk of radius x are, respectively, given by

$$\frac{V(0) - V(x)}{f(0) - f(x)} \text{ and } \frac{P(0) - P(x)}{f(0) - f(x)}. \tag{II.3}$$

Direct substitution of Eq. (II.2) in Eq. (II.3) will show that these two quantities are related by a straight-line equation with parameters a and b.

Thus, the required form of the function $\bar{v}(\bar{r})$ for $\bar{r} < \bar{r}_{min}$ is, in fact, linear and is a continuation of the linear form of $\bar{v}(\bar{r})$ for $\bar{r} > \bar{r}_{min}$.

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